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ABSTRACT

This teacher's guide contains instructional material for use in a module designed to help students learn to use a list as one way to organize information. Detailed teaching procedures are given for each of eight lessons as well as objectives, student pages and transparencies. An answer key is provided. The lessons address the following goals: (1) reading and organizing lists; (2) making headings and entries for lists; (3) solving problems where the list is the solution and problems where the solution is contained in the list; (4) performing calculations on list entries to solve; (5) solving problems when the list suggests relationships that help solution; and (6) solving problems using three diagrams. (MP)

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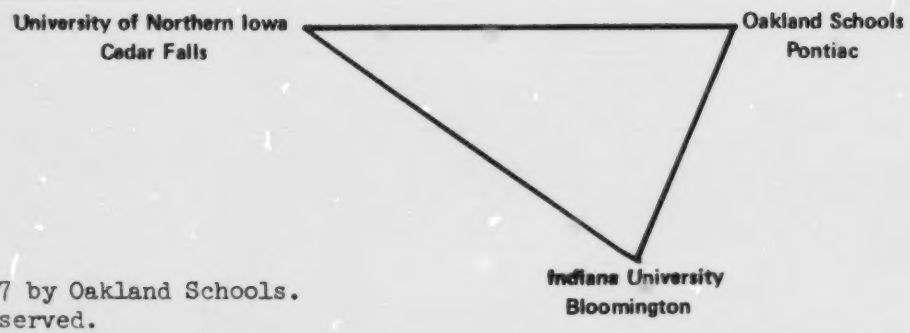
SE 026 919

Mathematical Problem Solving Project



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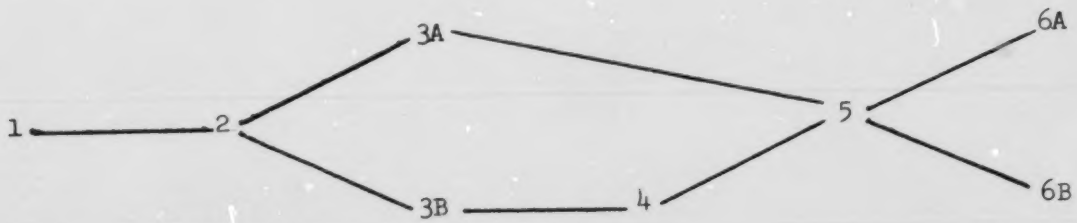
One characteristic of a good problem solver is the ability to organize information in ways which contribute to problem solution. Therefore, the purpose of this module is to help students learn to use a list as one way to organize information. In using an Organized List, the problem information is thought of categorically and is listed under headings which help keep the information organized. Specifically, a list is one framework within which information may be organized as part of the problem solving process.

At times the list itself is the solution itself. Sometimes the solution is contained as an entry in the list. There are some lists which exhibit qualities or relationships that give clues to problem solution even though the list does not contain the solution. The lessons address these uses having the following goals.

- Reading and organizing lists.
- Making headings and entries for lists.
- Solving problems where the list is the solution and problems where the solution is contained in the list.
- Performing calculations on list entries to solve.
- Solving problems when the list suggests relationships that help solution.
- Solving problems using tree diagrams.

Two tracks are suggested to provide for differences in grade level. Fourth grade students study lessons 1, 2, 3A, 5 and 6A while fifth and sixth grade students study lessons 1, 2, 3B, 4, 5, and 6B. The diagram indicates the lesson sequence.



LESSON 1

Objective: Given a complete list the student will
a) read and answer questions based on information in the list,
b) rearrange the list according to a specified criterion.

The teacher will demonstrate that a list can be used to solve problems.

Materials: Student Pages 2-6 and Transparencies 1-3.

Teaching Procedure:

Reading a List - Student Page 2.

1. Ask students what the chart represents.
2. Use transparency 1 to discuss the construction of the chart.
   * a) Each column has a heading. Discuss them.

     Britt played 45 minutes, scored 3 goals in 7 attempts, scored 4 out of 4 free throws, captured 4 rebounds, assisted in scoring 3 goals, had 2 personal fouls, and scored a total of 10 points.

   * b) Each player has his record listed in a single row. No information is repeated in the list.
   * c) The list is organized by column and row.

3. Ask questions below which students answer on page 2.

   (1) Who won the basketball game?
   (2) Who scored the most points for Michigan?
   Who scored the most points for Indiana?
   (3) Who was Michigan's third highest scorer?
   (4) Who had the most personal fouls for Indiana?
   (5) Who played the least time for Michigan?
   (6) Who made the most field goals for Indiana? How many?
   (7) Who tried the most field goals for Indiana? How many?

Discuss the questions as necessary.

* These statements are the elements of a complete organized list.

MICHIGAN – INDIANA

MICHIGAN

	Minutes	Field Goals	Free Throws	Rebounds	Personal Fouls	Points Scored
Britt	45	3-7	4-4	4	2	10
Robinson	40	2-6	2-2	6	2	6
Hubbard	38	4-14	4-4	15	5	12
Grote	37	6-10	4-5	5	5	16
Green	45	9-19	5-8	3	2	23
Hardy	5	0-0	0-1	2	0	0
Baxter	4	0-0	0-0	0	1	0
Bergen	7	0-0	0-0	1	1	0
Totals		24-56	19-24	42	18	67

INDIANA

	Minutes	Field Goals	Free Throws	Rebounds	Personal Fouls	Points Scored
May	45	11-30	5-7	6	4	27
Abernethy	29	0-4	0-0	5	2	0
Benson	45	9-17	3-4	15	2	21
Buckner	39	1-10	0-0	2	5	2
Wilkerson	28	1-8	0-0	6	3	2
Wisman	2	0-0	0-0	0	1	0
Valavicius	23	2-4	0-0	3	2	4
Radford	24	6-7	4-4	5	2	16
Crews	1	0-1	0-0	1	0	0
Totals		30-81	12-15	48	21	72

MICHIGAN

	MIN	FG	FT	REB	PF	PTS
BRITT	45	3-7	4-4	4	2	10
ROBINSON	40	2-6	2-2	6	2	6
HUBBARD	38	4-14	4-4	15	5	12
GROTE	37	6-10	4-5	5	5	16
GREEN	45	9-19	5-8	3	2	23
HARDY	5	0-0	0-1	2	0	0
BAXTER	4	0-0	0-0	0	1	0
BERGEN	7	0-0	0-0	1	1	0
TOTALS		24-56	19-24	42	18	67

LESSON 1

MICHIGAN – INDIANA

MICHIGAN

	Minutes	Field Goals	Free Throws	Rebounds	Personal Fouls	Points Scored
Britt	45	3-7	4-4	4	2	10
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Hardy	5	0-0	0-1	2	0	0
Baxter	4	0-0	0-0	0	1	0
Bergen	7	0-0	0-0	1	1	0
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	Minutes	Field Goals	Free Throws	Rebounds	Personal Fouls	Points Scored
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Buckner	39	1-10	0-0	2	5	2
Wilkerson	28	1-8	0-0	6	3	2
Wisman	2	0-0	0-0	0	1	0
Valavicius	23	2-4	0-0	3	2	4
Radford	24	6-7	4-4	5	2	16
Crews	1	0-1	0-0	1	0	0
Totals		30-81	12-15	48	21	72

1. _____
2. _____
   _____
3. _____
4. _____
5. _____
6. _____ ____
7. _____ ____

6 9

Rearranging a List - Student Page 3.

1. The basketball chart for Michigan should be rearranged by some of the criteria listed below.
   Each group can be given one criterion. Then different lists will be constructed.
   The lists on student page 2 are organized by the criterion "The order the players entered the game."

   Rearrange the List according to:

   Alphabetical, minutes played, field goals made, points scored, field goals attempted, free throws made,...

2. Ask questions which illustrate that some list arrangements lend themselves to easier answers than others.

   (a) Who was the fourth highest scorer?
   What list was most convenient for answering the question?
   (The list organized by points scored.)
   (b) Who played the third highest number of minutes?
   Which list was best?
   (c) Who made the least number of personal fouls?
   Which list was best?
   (d) Whose name appeared at the top of the list most times?
   (Poll the groups and tally on the chalkboard. It was Green.
   This could give a clue as to the most valuable player.)

Rearranging a List

MICHIGAN

Name	Minutes	Field Goals	Free Throws	Rebounds	Personal Fouls	Total Points

This list has been organized according to _____.

Answering Questions Using a List

1. The entries in a list can sometimes be used to answer questions.
2. Using transparency 1 show the answer to the question: Which player did the best job of scoring for the shots he took?

   (a) Reveal the per cent of completions and show that Grote ranked highest.
   (b) Explain that the field goal column was used, a calculation was made and it was found who scored at the highest rate.
   (c) Note: It is not intended that percent be mentioned; only that entries were used to obtain more information. Also, it is realized that scoring percent can be deceiving for it is the highest score that wins. Do not get into a discussion of the implications of this column of information.

Where Lists Come From

1. Students often think lists appear out of thin air.
2. Show Transparency 2, the game summary for Jacksboro High. Then show the game chart, Transparancy 3. Tell them that
   (1) "2's" show points scored.
   (2) 0 free throw attempts.
   (3) 0 free throws made
   (4) Personal Fouls made are indicated by an "X".
3. This is intended to be a brief indication to students that a list can put information in a more usable form.

TEAM JACKSBORO

COACH ROSY HAYES

DATE JAN 3, 1976

SCORER C.L. DEATON

RUNNING SCORE

TIMER ED GOFF

FIRST QUARTER SCORE 11 | FIRST HALF SCORE 27 | THIRD QUARTER SCORE 45 | FINAL SCORE 59

POS	QUARTERS PLAYED	PLAYER	NO.	FOULS	FIRST HALF 1ST QTR.	FIRST HALF 2ND QTR.	SECOND HALF 3RD QTR.	SECOND HALF 4TH QTR.	OVER-TIME	SUMMARY FG	FT	F	TP
C	1Q 2Q 4Q	DAVIS	11	P5	2	2 0 ●	2			3	1	4	7
F	1Q 2Q 3Q 4Q	WYNN	7	P4 P5	●	2 ●	●	2 0		2	3	3	7
F	3Q 4Q	WARLICK	22	P3 P4 P5	2, 2	0				2	0	2	4
G	1Q 2Q 3Q 4Q	MARTINEZ	17	P4 P5		2	2 0 2 ● 2	2 ● 2		6	3	3	15
C	2Q 3Q 4Q	HAMILTON	14	P2 P3 P4 P5	● 0					0	1	1	1
G	3Q	SCHARTz	6	P4 P5	2 ●	2 0 ●		2 ● 2		4	3	3	11
F	1Q 4Q	TANKERSLEY	21	P2 P3 P4 P5			2 ●			1	1	1	3
G	1Q 4Q	SLEDGE	18	P3 P4 P5		2 ● ●				1	2	2	4
G	1Q 2Q 3Q 4Q	VANCE	8	P1 P2 P3 P4 P5									
F	1Q 2Q 3Q 4Q	LOCKHART	27	P1 P2 P3 P4 P5									
C	1Q 2Q	GOLDBERG	13	P4 P5			2 ● 2	2		3	1	3	7
G	1Q 2Q 3Q 4Q	TRIMBLE	15	P1 P2 P3 P4 P5									
	1Q 2Q 3Q 4Q			P1 P2 P3 P4 P5									
	1Q 2Q 3Q 4Q			P1 P2 P3 P4 P5									

TIME OUTS 1ST Q	2ND Q	3RD Q	4TH Q
1	1	1	11

TEAM FOULS FIRST HALF 5 6 | SECOND HALF 5 6

FREE THROWS ATTEMPTED 21 | FREE THROWS MADE 15 | PERCENT 71

TEAM TOTALS	FG	FT	F	TP
	22	15	22	59

-17-

TEAM JACKSBORO

COACH ROSY HAYES

DATE JAN 3, 1976

SCORER C.L. DEATON

RUNNING SCORE

1	4	6	9	12	16	18	21	25	28	31	34	36	38	41	43	46	49	51	53	55	57	60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101
102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133
134	135	136																													

TIMER ED GOFF

FIRST QUARTER SCORE 11 | FIRST HALF SCORE 27 | THIRD QUARTER SCORE 45 | FINAL SCORE 59

QUARTERS PLAYED	PLAYER	NO.	FOULS	FIRST HALF 1ST QTR.	FIRST HALF 2ND QTR	SECOND HALF 3RD QTR	SECOND HALF 4TH QTR.	OVER-TIME	SUMMARY FG	FT	F	TP
1Q 2Q 4Q	DAVIS	11	P5	2	2 ⊗	2			3	1	4	7
1Q 2Q 3Q 4Q	WYNN	7	P4 P5	⊗	2 ⊗	⊗	2		2	3	3	7
1Q 2Q	WARLICK	22	P3 P4 P5	2, 2					2	0	2	4
2Q 3Q 4Q	MARTINEZ	17	P4 P5		2	2, 2, 2 ⊗	2, 2 ⊗		6	3	3	15
1Q 2Q 3Q 4Q	HAMILTON	14	P2 P3 P4 P5	⊗					0	1	1	1
1Q 2Q 3Q	SCHARTZ	6	P4 P5	2 ⊗	2 ⊗		2 2 ⊗		4	3	3	11
1Q 2Q 4Q	TANKERSLEY	21	P2 P3 P4 P5			2 ⊗			1	1	1	3
1Q 2Q 4Q	SLEDGE	18	P3 P4 P5		2 ⊗				1	2	2	4
1Q 2Q 3Q 4Q	VANCE	8	P1 P2 P3 P4 P5									
1Q 2Q 3Q 4Q	LOCKHART	27	P1 P2 P3 P4 P5									
1Q 2Q 3Q 4Q	GOLDBERG	13	P4 P5			2 ⊗ 2	2		3	1	3	7
1Q 2Q 3Q 4Q	TRIMBLE	15	P1 P2 P3 P4 P5									
1Q 2Q 3Q 4Q			P1 P2 P3 P4 P5									
1Q 2Q 3Q 4Q			P1 P2 P3 P4 P5									

TIME OUTS 1ST Q	2ND Q	3RD Q	4TH Q
1	1	1	11

TEAM FOULS FIRST HALF: X X X X 5 6 | SECOND HALF: X X X X 5 6

FREE THROWS ATTEMPTED 21 | FREE THROWS MADE 15

TEAM TOTALS	22	15	22	59
PERCENT	71			

TRANSPARENCY 2

16

17

JACKSBORO

	FG	FT	PF	TP
DAVIS	3	1-2	4	7
WYNN	2	3-4	3	7
WARLICK	2	0-1	2	4
SCHARTZ	4	3-4	3	11
MARTINEZ	6	3-3	3	15
HAMILTON	0	1-2	1	1
TANKERSLEY	1	1-1	1	3
SLEDGE	1	2-2	2	4
GOLDBERG	3	1-1	3	7
TOTALS	22	15-21	22	59

Organizing Lists - Students pages 4-6.

Small groups may work on these pages.
Observe and discuss lists with the students.

Look over the list of cities and their populations.

U.S. Cities	Population
Baltimore	905 787
Chicago	3 369 357
Cleveland	750 879
Dallas	844 401
Detroit	1 513 601
Houston	1 232 802
Los Angeles	2 809 813
New York	7 895 563
Philadelphia	1 949 996
Washington, D.C.	756 510

1. Organize the list according to population, largest to smallest.

City	Population

2. What is the 8th largest city?

3. Where does Detroit rank on the list?

4. Which city has a population nearest to Houston?

5. Which two cities together have about the same population as Los Angeles?

Outstanding Football Players

I

Player	Total Yards Rushing	Average Yards Per Carry
Brown	12 312	5.2
Johnson	6 803	4.3
Kelly	7 274	4.2
Perkins	6 217	4.1
Perry	9 723	5.0
Taylor	8 597	4.4
Willard	6 105	3.8

1. How is the above list organized?

2. Reorganize the list in the chart below. Use total rushing yardage from greatest to least.

II

Player	Total Yards Rushing	Average Yards Per Carry

3. Which player ranks in the middle of the reorganized list?

4. How many players have rushed more than 7 000 yards?

5. Which player ranked 5th in <u>average yards per carry</u>?

6. Was the reorganized list (II) very helpful in answering question 5?

7. Reorganize the list according to average yards per carry, from highest to lowest.

III

Player	Total Yards Rushing	Average Yards Per Carry

8. Which player ranked 4th in average yards per carry?

9. Which list, II or III, is most helpful in answering question 5?

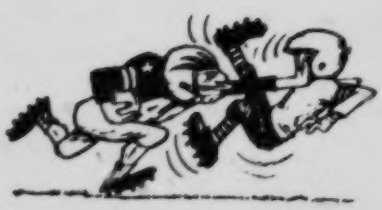
10. Which three players are ranked in the middle for average yards per carry?

11. Which players are listed in the middle for each of the lists I, II, and III?

    Are they the same person?

12. Which players rank in the top 3 in lists II and III?

    Are they the same people?



LESSON 2

Objective: Given a partial list and a problem situation the student will complete the list by writing missing headings and/or missing entries.

Given a set of raw data, the student will make a complete list. *

Materials: Student pages 8-10.

Teaching Procedures: Following are two examples (A and B) which the teacher is to model.

* Remember, a complete list has 1) headings, 2) entries by single row, 3) no information is repeated, 4) the list is organized by column and row.

1. Tell the students, "I have change in my pocket for 50¢, and I do not have any pennies included. I wonder what the possibilities are?"
2. After a suggestion or two is made ask the question, "What are the possible coins?" They are quarters, nickels, and dimes Tell them you will make a list of all the possibilities. Set up the list on the chalkboard with the first three entries:

Quarters	Dimes	Nickels
2	0	0
1	0	5
1	1	3
1		

Ask, "What is another possibility using one quarter?"
(2 dimes and 1 nickel)
Ask the students for more entries to complete the list.

The complete list:

Quarters	Dimes	Nickels
2	0	0
1	0	5
1	1	3
1	2	1
0	0	10
0	1	8
0	2	6
0	3	4
0	4	2
0	5	0

3. Point out that:
   a) the columns are labeled to help keep track of what the entries mean.
   b) the list contains 10 rows which show the ways change can be made for 50¢.
   c) no rows are repeated.
   d) the list is the solution to the problem.
4. Ask some questions:
   a) how many ways use no dimes?
   b) which coin is used in more possibilities than the other two coins?
   c) how many ways can change for 50¢ be made using just 4 coins?
   d) which way uses the highest number of coins?
   e) can change be made using just one kind of coin? How many ways?
5. Point out that the list can contain answers to questions. Sometimes the whole list is a problem solution; sometimes only parts of a list are used.

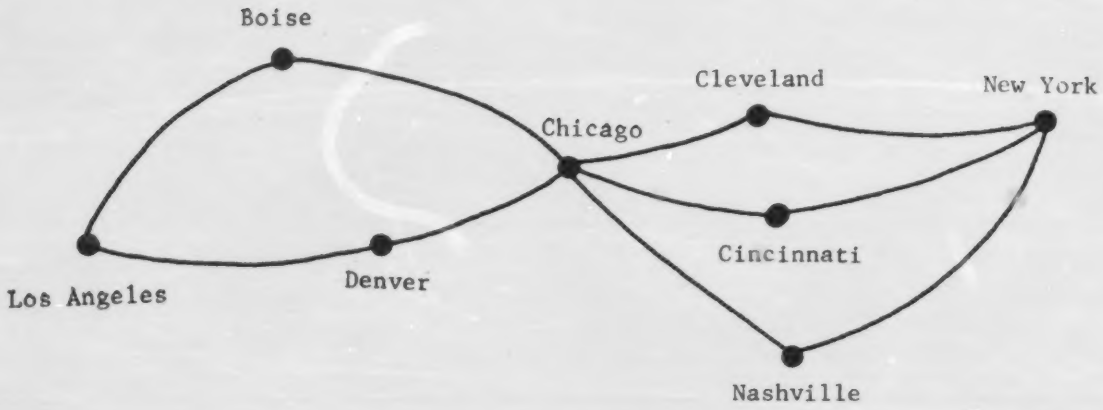
1. Say to students, "I will ask some of you about when you were born." Ask a student for birthday information and he should tell you the month, day, year, as well as name. Write a sentence like 'Jane Doe was born on January 15 in the year 1964.'
   Ask the students how a list should be organized to record this information.
   The columns should be: name, day, month, year. The point is to organize the information using headings. Make a list using entries supplied by several students.
2. Ask how the list might be reorganized (by day, month, or year, alphabetical by name.)

Student Pages 8-10

1. Assign the student pages for group work.
2. These pages present situations where headings, entries, or both are missing. It will be possible to show where the list is the solution as well as where the list contains the solution.

FOLLOWING LESSON 2, FOURTH GRADE STUDENTS SHOULD GO TO LESSON 3A.
FOLLOWING LESSON 2, FIFTH AND SIXTH GRADE STUDENTS SHOULD GO TO LESSON 3B.

LESSON 2



One way to travel from Los Angeles to New York is to go through Boise, Chicago and Cleveland.

Another way is to go through Boise, Chicago and Cincinnati.

1. Make a complete list of the ways to go from Los Angeles to New York on this map.

1st Stop	2nd Stop	3rd Stop
Boise	Chicago	Cleveland
Boise	Chicago	Cincinnati

2. How many of the possible routes go through Denver?

3. How many of the routes go through Nashville?

1. Mrs. Lowe has 3 lunch sacks and only 2 doughnuts. Complete the list.

First Sack	Second Sack	Third Sack

a. How many ways can she put 2 doughnuts in the lunch sacks?

b. How many ways can the first sack be left empty?

c. How many ways can the first sack contain 1 doughnut?

d. How many ways can the first sack contain 2 doughnuts?

2. Some students gathered information about some of their teachers. Then the information was put in a list but column headings were left off.

Put in the headings for the students.

Mr. B.	4th	30	November	1940	6'2"	195	Vega
Mr. C.	4th	15	October	1935	5'10"	175	Corvette
Miss J.	5th	5	July	1944	5'4"	115	Duster
Mr. M.	4th	4	March	1925	5'11"	185	LTD
Mrs. N.	6th	21	March	1947	5'6"	130	VW
Mrs. R.	6th	23	August	1936	5'2"	110	Ford
Miss S.	5th	17	December	1953	5'2"	115	Chrysler

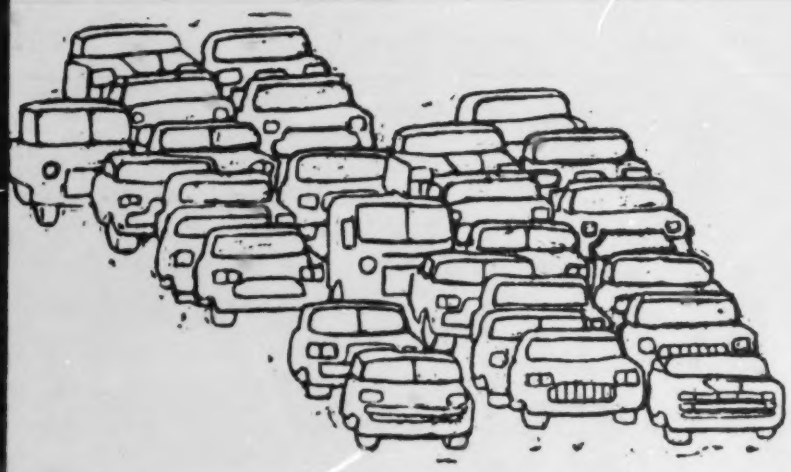
1. Six of the tallest buildings in the U.S. are in New York and Chicago. Organize a list of these buildings.

New York:	World Trade Center	1350 feet and 110 stories
	Empire State	1250 feet and 102 stories
	Chrysler	1046 feet and 77 stories
Chicago:	John Hancock	100 stories and 1127 feet
	Standard Oil	80 stories and 1136 feet
	Sears	110 stories and 1454 feet

a. I organized my list according to _____.

b. Did the list turn out to be organized in order for both columns?

2. James runs two parking lots called Ace Parking and Safe Parking. He parks cars for people who eat at a small restaurant nearby. Make a list to show the ways he could park 9 cars between the two lots.



LESSON 3A

(FOURTH GRADE STUDENTS)

Objective: Given a problem situation and a list, the student will answer questions about the problem situation.

Given a problem situation, the student will make a list to solve.

Materials: Student Pages 12-15.

Teaching Procedure: Lesson 3A is appropriate for fourth grade level students.

Solving Problems Using a List -Student Pages 12-15.

1. Use student page 12 with students. They are to read the situation and list in order to answer questions.
2. Student page 13 illustrates that the list itself could be the solution or a specific entry could indicate the solution.
3. Pages 14-15 are problem situations in which students are to make lists to solve. Group work is appropriate for these pages.

FOLLOWING LESSON 3A, FOURTH GRADE STUDENTS SHOULD GO TO LESSON 5.

LESSON 3A

Population records help us see growth and changes in cities. The list below shows the records for some U.S. cities. Answer the questions using the information in the list.

How Cities Grew - Population

	1970	1960	1950	1900	1850
New York	7 895 563	7 781 984	7 891 957	3 437 202	696 115
Chicago	3 369 357	3 550 404	3 620 962	1 698 575	29 963
Los Angeles	2 809 813	2 479 015	1 970 358	102 479	1 610
Detroit	1 513 601	1 670 144	1 849 568	285 704	21 019
Houston	1 232 802	938 219	596 163	44 633	2 396
Boston	641 071	697 197	801 444	560 892	136 881

1. How is this list organized? By _____

2. Which city had the 5th largest population in 1970?

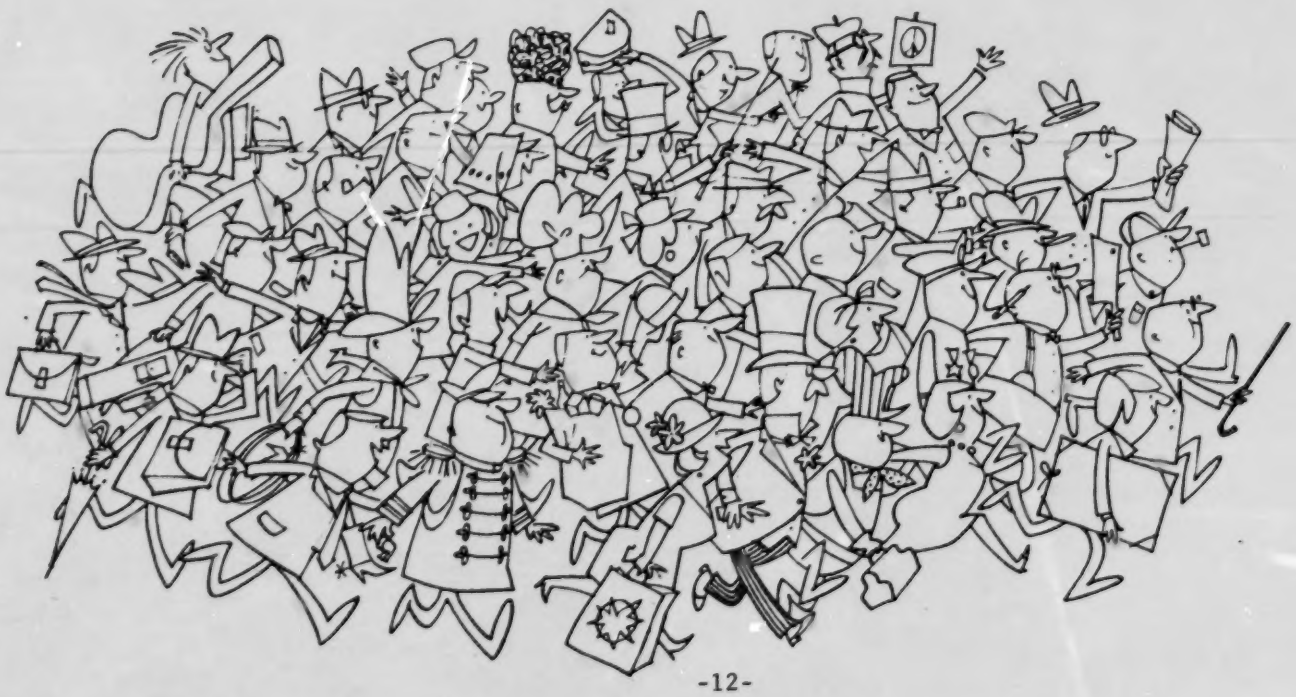
3. Which city had the smallest population in 1850?

4. Which city gained the most people from 1850 to 1970?

5. Which city gained the least population from 1850 to 1970?

6. Why do you think Los Angeles and Houston had such small populations in 1850?

7. Which cities show a decrease in population from 1960 to 1970?



Show the ways that you could make change for 50¢ using nickels and dimes.

1. Is the solution to the problem a complete list or an entry in the list?

2. How many ways can you make change for 50¢ using nickels and dimes?

3. If exactly eight coins were used, what would they be?

4. A nickel weighs 5 grams and a dime weighs 2.2 grams. To find which way of making change would be heaviest in your pocket, what new columns would you add to your list?

5. Go back to your list, put in the new columns and find which way of making change would be the heaviest.

1. List all the ways to arrange the letters of the word "EAT."

   a. How many ways can the letters be arranged?

   b. How many of the new words are real words found in a dictionary?

2. Go ask 10 of your classmates for some information: their name, age, their birthdate, their favorite singer, which subject in school they like best, favorite sport.

   Make an organized list of the information.

   a. Who is the oldest?

   b. Who is 6th oldest?

   c. Who are the 3 youngest girls?

   d. Which subject is liked by the most students?

   e. Did one singer get the most votes?

   f. What did you find out about favorite sports?

123456789

Make a list to show the ways three numbers can be added to equal 10.
A number may be used more than once.

1. How many ways use even numbers only?

2. Which number(s) in the list gets used the most?

3. Which number(s) in the list gets used the least?

LESSON 3B

(FIFTH AND SIXTH GRADE STUDENTS)

Objective: The student will solve problems using a list given a problem situation in which

a) the list is the solution,

b) the list contains the solution as specific entries

Materials: Student pages 17-18.

Teaching Procedure: Following are three examples (A, B and C) which the teacher is to model.

The List as a Solution

1. Discuss with the students that sometimes the solution to a problem is the list one makes. Sometimes the list is re-organized to make its application easier.

   Example: When the student's mother makes a shopping list for the supermarket, the list of items she has is the solution.

   The list might be improved by rearranging the items by the aisle in the store where the item is found so she would not return to the same place twice. Such categories might be dairy products, meat, fresh produce, frozen vegetables, soaps, bakery, etc.

2. The problem in Lesson 2, making change for 50¢, is an example of a list being the solution.
   Review the problem: Find the ways to make change for 50¢, excluding pennies. The kinds of coins were found, column headings were written, the ten rows of entries were made. The list is the solution.

3. Model this problem: The coach ordered three dozen baseballs for the coming season. Each ball is packed in its own small box which is a cube (36 balls = 36 cubes). In what different shaped boxes (rectangular solid) might the 36 baseballs be shipped? The shape of the shipping box is determined by the number of individual cube boxes (n=36) making up its length, height and width.

Example: one shipping box might be of size

L = 2 boxes long
W = 3 boxes wide
H = 6 boxes high

Question: Will the solution be the complete list or one of the entries in the list? (Complete list)

Solution:

(a) The box is measured by length, width, and height. These are the three factors to keep tract of, so a three column list is appropriate.

(b) Set up the list with the students:

L	W	H

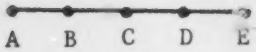
(c) Then make entries to get the list:

L	W	H
1	1	36
1	2	18
1	3	12
1	4	9
1	6	6
2	2	9
2	3	6
3	3	4

(d) Questions:
   (1) How many different shapes are possible? (8)
   (2) Which box is the tallest? (1,1,36)
   (3) Which shape is most like a cube? (3,3,4)
   (4) How many boxes would look square when observed from one side? (4)

4. Use Student Page 17 with the students working in groups.

LESSON 3B

1. List the possible segments that can be made from 

2. List the different ways to write a 3-digit number using 5,7, and 9.

Examples are: 579, 795, and 597

TEACHING PROCEDURE FOR EXAMPLE B

The List Contains the Solution

1. Discuss that some lists are made so the solution or solutions can be found within the list. Point out that some problems have more than one solution. Some of the questions about the basketball game were specific entries and not the entire list.
2. Model the problem: Two students add their ages and get 18. When subtracted, the result is 2. What are the ages?
   (a) Ask questions:

   (1) What are the key numbers? (18,2)
   (2) How is 18 found? (by adding ages)
   (3) How is 2 found? (by subtracting ages)
   (4) Therefore, what are the key operations? (add,subtract)
   (5) What will the solution tell us? (The ages of the two students)
   (6) How should the list be set up?

   (b) Pairs of ages will be tried to see if their sum is 18 and their difference is 2.

The list:

1st Student	2nd student	Sum	Difference
17	1	18	16
16	2	18	14
15	3	18	12
14	4	18	10
13	5	18	8
12	6	18	6
11	7	18	4
10	8	18	2 *
9	9	18	0
	Reversals will now occur in the list.		

* is the solution.

   (c) Point out that the age entries were used to calculate the sum and difference. The Sum was used to check for the correct age entry. The difference was used to determine when the solution was found.

   (d) Also point out that a complete list (using specifically the criterion of recording all entries) is not needed here. Students might have ESTIMATED the correct answer and then worked up or down from there. The point is, the list helped them organize and systematize their trials.

The List Contains More Than One Solution

Al's father is off work every sixth day during the month beginning with day 6. Al is off work from the supermarket every fourth day beginning with day 4. When will Al and his father both be off work?

1. Set up the column headings:

Al's Father days off	Al days off

2. Fill in the table:

Al's Father days off	Al days off
6	4
12	8
18	12
24	16
30	20
	24
	28

3. Al and his father are both off work on the 12th and the 24th.

Point out that the list shows <u>two</u> solutions. A list might contain more than one solution, and that is why a complete list is important

4. The lesson objective should be summarized.

   There are situations where the complete list is the solution. For example, packing baseballs and mom's grocery list organized by aisles. Sometimes the list contains the solution as an entry. For some students a complete list will not be required because they will estimate the solution and refine their estimates resulting in only a few entries in the list. However, the list provides a way to organize their thinking.

Note to Teacher:

At this point the student should begin to discriminate between those situations requiring a complete list because it is the solution and those situations when a list is used to organize the information so the solution may be found.

Use Student Page 18 with the students working in groups.

What is the fewest number of students at a crafts class if red paint is shared in groups of 2, green ribbon in groups of 5, and the silver sparkle is shared in groups of 6?

A brother and sister found that the product of their ages was 72. Also the sum of their ages was 18. Use a list to find the two ages.

LESSON 4

(FIFTH AND SIXTH GRADE STUDENTS)

Objective: Given a problem situation with or without the list, the student will answer questions requiring calculations of entries in the list.

Given a problem situation, the student will inspect the list to find relationships that lead to solution.

Materials: Student Pages 21-24

Teaching Procedure:

Review with students that some calculations were made when the basketball game summary was discussed in Lesson 1. Shooting percent was found by dividing field goals made by field goals attempted.

Also, in Lesson 3B the problem of finding student ages that add to 18 and have a difference of 2 required calculation of list entries.

Following are two teacher directed activities.

Using Lists to Gain More Information - Student Page 21

1. Discuss the problem situation with the students.
2. Have the students work in groups answering questions which require calculations of list entries.

LESSON 4

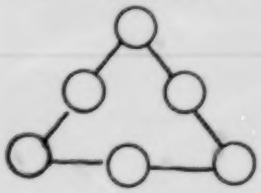
"Rafael, make a list using your last 10 homework assignments. I will pay you 5¢ times the number right minus number wrong for each assignment.

Assignment	Number of problems in assignment	Number correct			Payoff
1	10	9			
2	8	6			
3	15	12			
4	7	7			
5	6	6			
6	10	8			
7	12	9			
8	15	11			
9	20	15			
10	16	12			

How much did Rafael earn altogether?

1. Discuss the problem situation. They are to find an arrangement of six numbers that can be placed in the circles so the sums of the three sides are all 9.

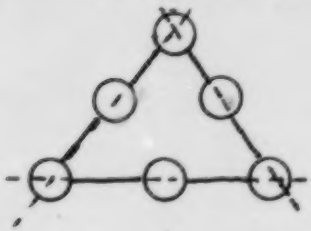
The problem: Place each of the numbers 1,2,3,4,5,6 in the cells so the sum of the numbers along each side is 9.



(a) Tell the students to make a list of all the triples whose sum is 9.

1st number	2nd number	3rd number	Sum
6	1	2	9
5	1	3	9
4	2	3	9

(b) Have students lightly draw lines along the triangles' sides showing the cells whose numbers are added. You may have to illustrate one side.



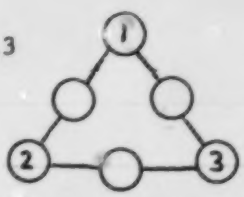
Have students note that the vertices are used in two sums.

Also look at the list.

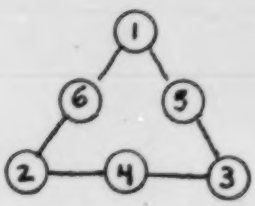
1st	2nd	3rd
6	1	2
5	1	3
4	2	3

The numbers 1,2,3 appear in two sums. Ask, "Where should 1,2,3 go?" (at the vertices.)

(c) After placing 1,2,3

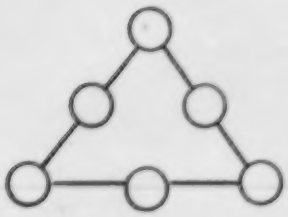


the other numbers are easily placed.



(d) Make the point: The list for this problem gave clues to the solution because of the number of times certain numbers appeared. The list was not the solution, nor did the solution appear in the list.

Place each of the numbers 1,2,3,4,5,6 in the cells so the sum of the numbers along each side is 9.



Make a list:

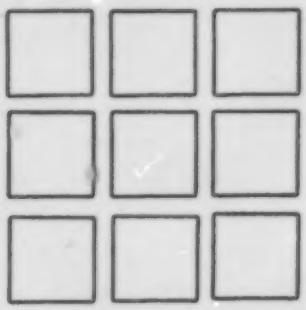
1st number	2nd number	3rd number	sum

Student Pages 23-24

1. Have the students work in groups on problems which require entry calculations as well as giving clues to the problem solution.
2. Observe their work and discuss the problems with them.

1. Using 5 darts, how can Don score 31 points?

2. Place the numerals 2 through 10 in squares so the sum of three squares in any direction is 18.



Balloons cost 3¢ each. Whistles cost 5¢ each.
The most you can buy of each is 4.

Make a list to show the possibilities.

1. If you spent 8¢, what did you buy?

2. What can be bought with 17¢?

3. Could exactly 25¢ be spent?

4. What will 21¢ buy?

5. What is the greatest amount you could spend?

LESSON 5

The teacher will demonstrate the "tree diagram" as a type of list.

Objective: Given a problem situation and

a) a complete tree diagram, the student will be able to answer questions using the diagram.
b) a partial tree diagram, the student will complete the diagram.
c) instructions to make a tree diagram, the student will construct the diagram.

Materials: Student Pages 26-28

Transparency 4 -- Hexahedron

Hexahedron model made from pattern in student booklet, page 26a

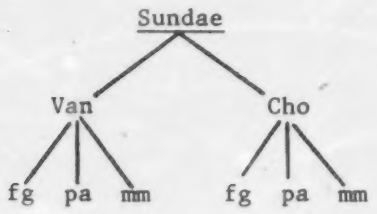
Teaching Procedure: Following are two examples (A and B) which the teacher is to model.

Tree Diagrams

While the column-type list is likely the most used style, there are other forms of lists such as tables, matrices, and tree diagrams. In this lesson, the tree diagram will be presented.

1. Present the problem: "You have two kinds of ice cream, vanilla and chocolate. There are three toppings, fudge, pineapple, and marshmallow. You may choose one ice cream and one topping. What are the possible sundaes you can choose?

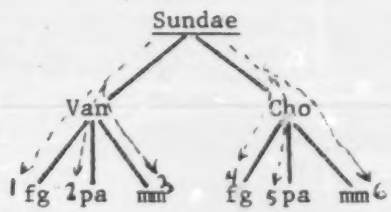
   (a) Tell the students that a list or a tree diagram can be used. The choice here is a tree diagram.



2 choices of ice cream

with each ice cream there are three topping choices.

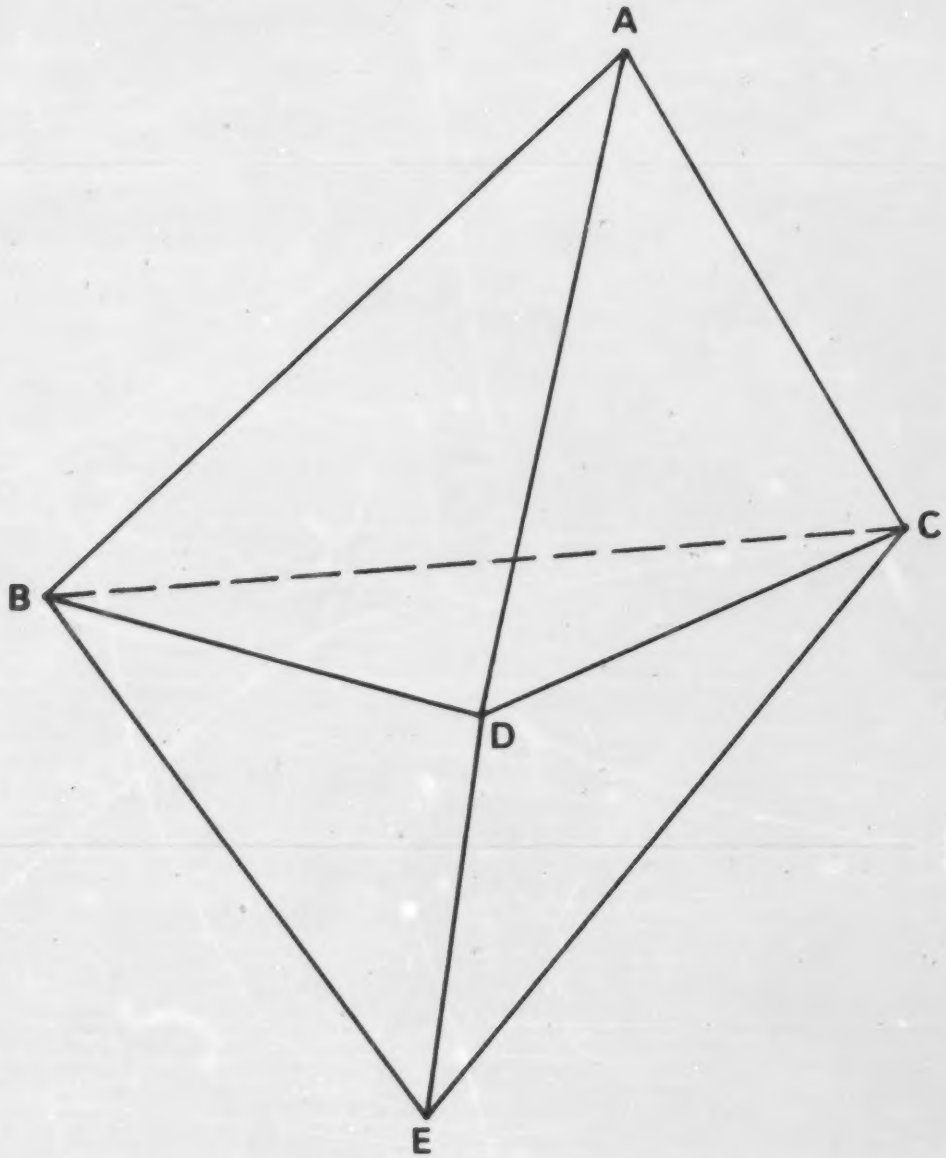
The number of sundaes can be counted:



Six ways!

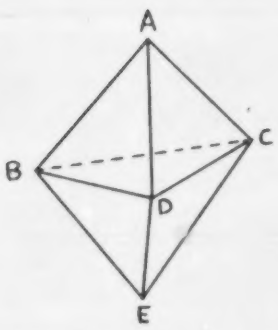
(b) The solution is the same using a list.

Ice cream	Topping
Vanilla	Fudge
Vanilla	Pineapple
Vanilla	Marshmallow
Chocolate	Fudge
Chocolate	Pineapple
Chocolate	Marshmallow

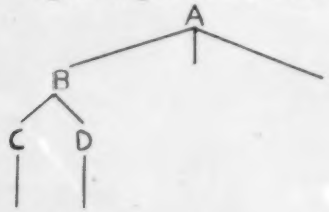


TRANSPARENCY 4

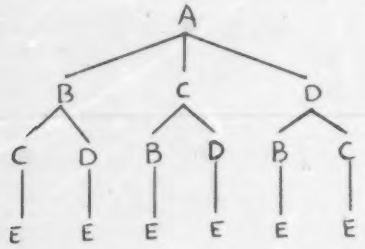
2. Use Student Page 26



(a) Discuss the problem situation and list all the three-legged trips from A to E.
(b) Show students one trip: A to B to D to E; three segments are traveled.
(c) "If the first leg is to B, what are the possible second legs?" (to C or to D)
(d) "Where do you go for the third leg?" (to E)
(e) Draw the beginning of the tree diagram on the chalkboard.



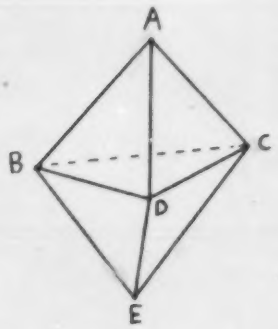
(f) Have students copy the beginning of the tree, and complete the left branch. (A-B-C-E)
(g) Ask for another possible first leg. (C for instance, and show them where to place C.)
(h) Have students complete the tree.



There are 6 trips.

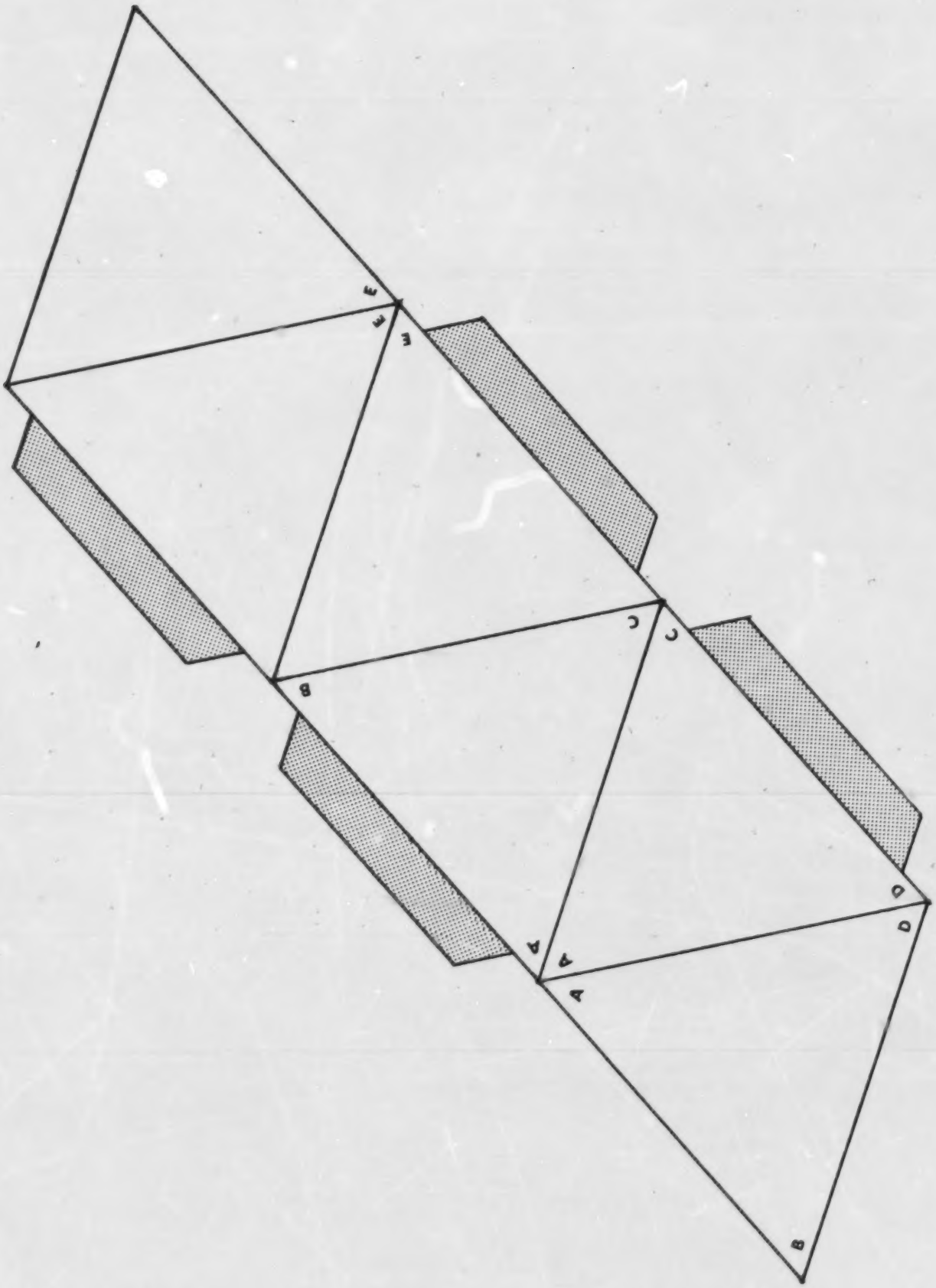
LESSON 5

Making a Tree Diagram



A spider crawls from point A to point E.
He crawls along <u>three</u> segments each time.

What are the 3-legged trips the spider could travel?


E
E
E
C
C
C
B
B
B
D
D
D
A
A
A

Solving Problems - Student Pages 27-29 (FOURTH GRADE STUDENTS SHOULD GO TO LESSON 6A AFTER ATTEMPTING PAGE 28.)

1. Have student groups solve the problems.
2. Discuss the problems with students.

FOLLOWING LESSON 5, FIFTH AND SIXTH GRADE STUDENTS SHOULD GO TO LESSON 6B.

Make a tree diagram to solve the problem.

Show the ways that Vicki, Mary, and Cathy could stand in line to buy ice cream.

At the fair the choices are hamburg, hot dog, and chili dog. You don't want to get bored making them so you decide to change the order you make them.

On each order you put mustard, catsup, and onions.

Draw a tree diagram to show the different ways you can prepare the food.

A paper sack contains 2 red and 2 green marbles.
You are to take out one marble at a time.

Draw a tree diagram to show the ways the marbles can be taken from the sack.

LESSON 6A

(FOURTH GRADE STUDENTS)

Objective: The teacher will review the previous lesson objectives. The student will solve problems similar to the resource deck in preparation for its use.

Materials: Student Pages 31-32.

Teaching Procedure:

1. Review the purpose and form of a list.

   (a) Lists help keep information in order.
   (b) Lists are sometimes the solution to problems.
   (c) Other times a list may contain the solution as an entry.
   (d) Since a list has information organized, it may give a clue to the solution of a problem.
   (e) After reading a problem for which a list seems sensible, information is grouped by categories.
   (f) Lists have headings so information is kept straight.
   (g) A careful approach to making a list can help assure that no possibilities are left out.

2. Use this problem to illustrate making a list.

   (a) If you bought 7 candy bars and your choices were Big Bar and Crunch Candy, what are the possible purchases?

heading → entries →

Big Bar	Crunch Candy
0	7
1	6
2	5
3	4
4	3
5	2
6	1
7	0

   (b) If Big Bars cost 10¢ each while Crunch Candy bars cost 15¢ each, and you spent 85¢, how many of each did you buy?

Another column is needed

Big Bar 10¢	Crunch Candy 15¢	Total cost ¢	
0	7	105	
1	6	100	
2	5	95	
3	4	90	
4	3	85	← 4 Big Bars, 3 Crunch Bars
5	2	80	
6	1	75	
7	0	70	

Solving Problems - Student Pages 31-32.

1. Have students work in groups
2. After problems on pages 31-32 are finished, the students may select problems from the resource deck.

LESSON 6A

1. Show the ways you can put 3 coins in 3 piggy banks.

3	0	0
2	1	0

2. Here are the heights in centimeters of nine 11 year old boys.

134	147	152
160	155	128
152	145	162

If all the boys stood in line from the tallest to shortest, how tall would the middle one be?

1. A chair maker has 7 arms, 18 legs, 6 backs, and 8 seats.

   How many whole chairs can he make?

2. When you bought milk, the clerk gave you 42¢ in change.

   Find the possible ways the clerk could have given you the change if <u>only 2</u> pennies are used.

LESSON 6B

(FIFTH AND SIXTH GRADE STUDENTS)

Objective: Given a problem situation, the student will be able to solve the problem using both a "column" list and a "tree diagram." The student will solve problems similar to the resource deck in preparation for its use.

Materials: Student Pages 34-38.

Teaching Procedure:

Using Column Lists and Tree Diagrams - Student Pages 34-36.

1. Have students work in groups to solve the problems.
2. Discuss with students solutions to the problems.

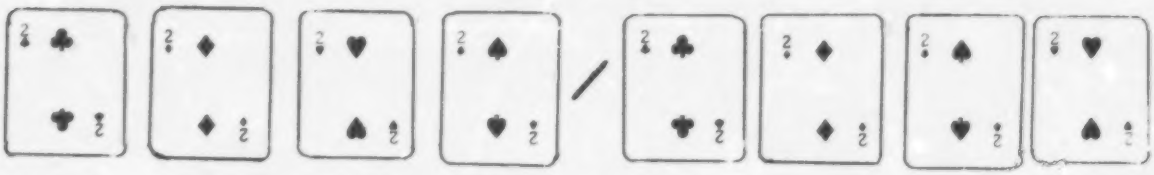
LESSON 6B

Your father has to go on a business trip to Chicago, Denver, and San Francisco. He wondered how many different routes he could go from home and visit each city.

Make both a column list and a tree diagram to show him the ways.

Four cards lay face down on a table: club, diamond, heart, spade.

If they are turned over, there are many orders for the cards. Two ways are shown:



Use both a tree diagram and a list to show how many ways the four cards could be turned up.

Tree Diagram for Cards

List for Cards

Solving More Problems - Student Page 37-38.

1. Students should work in groups.
2. When the problems are solved and checked, the students should select problems from the resource deck.

1. Make a list to show the ways 9 people can be placed in 3 groups so there is a different number of people in each group.

2. Find out your classmates favorite sports. Organize your findings so you can answer these questions.

   a. What sport is favored by girls?

   b. What sport is favored by boys?

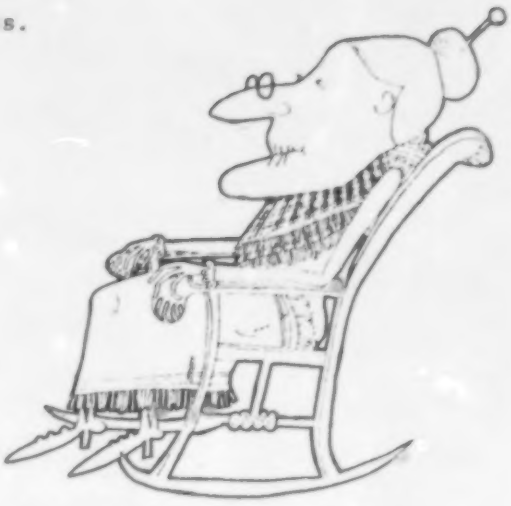
   c. Would you say boys and girls enjoy the same sports?

Grandma gave each of her grandchildren 25 cents. Each grandchild received the 25 cents in a different way.

Mike got 1 dime and 3 nickels, Joy got 25 pennies.

There are as many grandchildren as there are different ways to give 25 cents.

List the ways Grandma gave 25¢ to find the number of grandchildren.



ANSWER KEY

ORGANIZED LIST

Lesson 1

page 2 1. Indiana 2. Green, May 3. Hubbard
4. Buckner 5. Baxter 6. May, 11
7. May, 30

page 3(T4) There are 6 ways to organize the list.
2a. Britt 2b. Robinson 2c. Hardy 2d. Green

page 4 1.

City	Population
New York	7,895,563
Chicago	3,369,357
Los Angeles	2,809,813
Philadelphia	1,949,996
Detroit	1,513,601
Houston	1,232,802
Baltimore	905,787
Dallas	844,401
Washington D.C.	756,510
Cleveland	750,879

2. Dallas 3. 5th
4. Detroit 5. Houston and Detroit: 2,746,403

page 5-6 1. Alphabetical

II

2.

Player	Total Yards	Average
Brown	12,312	5.2
Perry	9,723	5.0
Taylor	8,597	4.4
Kelly	7,274	4.2
Johnson	6,803	4.3
Perkins	6,217	4.1
Willard	6,105	3.8

3. Kelly 4. 4 5. 4.2 6. No

7.

III

Player	Total Yards	Average
Brown	12,312	5.2
Perry	9,723	5.0
Taylor	8,597	4.4
Johnson	6,803	4.3
Kelly	7,274	4.2
Perkins	6,217	4.1
Willard	6,105	3.8

8. Johnson 9. III 10. Taylor, Johnson, Kelly

11. Perkins I, Kelly II, Johnson III No

12. Brown, Perry, Taylor II
Brown, Perry, Taylor III Yes

Lesson 2

page 8 1.

1st stop	2nd stop	3rd stop
Boise	Chicago	Cleveland
Boise	Chicago	Cincinnati
Boise	Chicago	Nashville
Denver	Chicago	Cleveland
Denver	Chicago	Cincinnati
Denver	Chicago	Nashville

2. 3 3. 2

page 9 1.

First sack	Second sack	Third sack
◎ ◎		
◎	◎	
	◎ ◎	
	◎	◎
		◎ ◎
		◎
◎		

a. 6 b. 3 c. 2 d. 1

page 9 2.

Name	Grade Teaches	Birth day	Birth Month	Birth Year	Height	Weight	Car

page 10 1.

Building Name	Height	Stories
Sears	1454	110
World Trade Center	1350	110
Empire State	1250	102
Standard Oil	1136	80
John Hancock	1127	100
Chrysler	1046	77

page 10 2.

Ace	Safe
0	9
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1
9	0

Lesson 3A page 12

1. By size
2. Houston
3. Los Angeles
4. New York
5. Boston
6. Before Westward Expansion
7. Chicago, Detroit, Boston

page 13

N	D
0	5
2	4
4	3
6	2
8	1
10	0

1. Complete List
2. 6 3. 6n, 2d
4. Weight of nickels
   Weight of dimes

5.

N	D	Weight nickels	Weight dimes	Total Weight
0	5	0	11.0	11.0
2	4	10	8.8	18.8
4	3	20	6.6	26.6
6	2	30	4.4	34.4
8	1	40	2.2	42.2
10	0	50	0	50.0

Heaviest

10 nickels
0 dimes
50 gm

page 14 1.

1st letter	2nd letter	3rd letter
E	A	T
E	T	A
A	E	T
A	T	E
T	E	A
T	A	E

1. 6

2. 3 EAT
   ATE
   TEA

page 14 2. Answers will vary for the classroom survey.

page 15

1st number	2nd number	3rd number
8	1	1
7	1	2
6	1	3
6	2	2
5	1	4
5	2	3
4	2	4
4	3	3

1. 2
2. 1,2
3. 8,7

Lesson 3B page 17 1.

LENGTH

1	2	3	4
AB	AC	AD	AE
BC	BD	BE	
CD	CE		
DE			

page 17 2.

1st digit	2nd digit	3rd digit
5	7	9
5	9	7
7	5	9
7	9	5
9	5	7
9	7	5

page 18

Red	Green	Silver
2	5	6
4	10	12
6	15	18
8	20	24
10	25	30
12	30	36
14	35	
16	40	
18		
20		
22		
24		
26		
28		
30		

30 students

page 19

Brother	Sister	Product	Sum	
1	72	72	73	
2	36	72	38	
3	24	72	27	
4	18	72	22	
6	12	72	18	*
8	9	72	17	

The ages are 6 and 12. It is possible that the sister is 6 and brother is 12.

page 21

Assign.	Total Problems	Number Correct	Number Wrong	Difference	Payoff
1	10	9	1	8	40
2	8	6	2	4	20
3	15	12	3	9	45
4	7	7	0	7	35
5	6	6	0	6	30
6	10	8	2	6	30
7	12	9	3	6	30
8	15	11	4	7	35
9	20	15	5	10	50
10	16	12	4	8	40

Total earned: $3.55

page 22

Completed in class discussion

page 23

1.

SCORE		
4	7	9
4	7	9
8	14	18
12	21	27
16	28	36
20	35	45

2 darts in 4:	8
2 darts in 7:	14
1 dart in 9:	9
	31

page 23 2.

1st number	2nd number	3rd number	sum
10	6	2	18
10	5	3	18
9	7	2	18
9	6	3	18
9	5	4	18
8	7	3	18
8	6	4	18
7	6	5	18

9	2	7
4	6	8
5	10	3

page 24

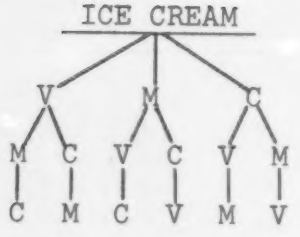
Balloon	Cost	Whistles	Cost
1	3¢	1	5¢
2	6¢	2	10¢
3	9¢	3	15¢
4	12¢	4	20¢

1. 1 Balloon and 1 Whistle
2. 4 Balloons and 1 Whistle
3. NO
4. 2 Balloons and 3 Whistle
5. 32¢

LESSON 5

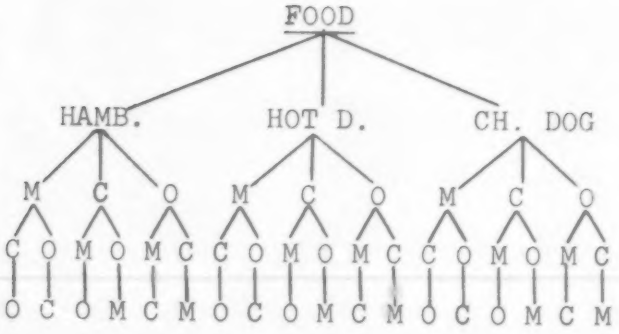
page 26 Problem is worked as class discussion.

page 27



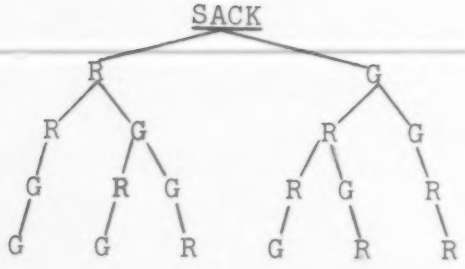
6 ways

page 28



18 ways

page 29



6 ways

LESSON 6A

page 31 1.

B1	B2	B3
3	0	0
0	3	0
0	0	3
2	1	0
2	0	1
1	2	0
1	0	2
1	1	1
0	2	1
0	1	2

page 31 2.

Heights in order

162
160
155
152
152 152 cm
147
145
134
128

page 32 1.

Chairs	Arms	Legs	Backs	Seats
1	2	4	1	1
2	4	8	2	2
3	(6)	12	3	3 *
4	8	(16)	4	4

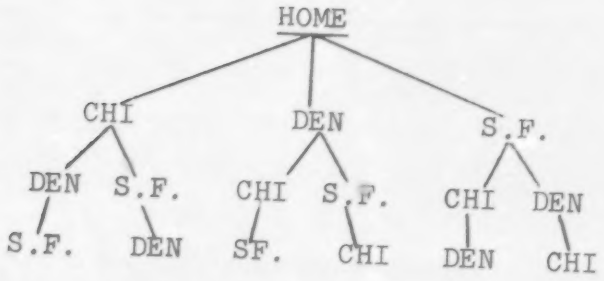
3 CHAIRS

page 32 2.

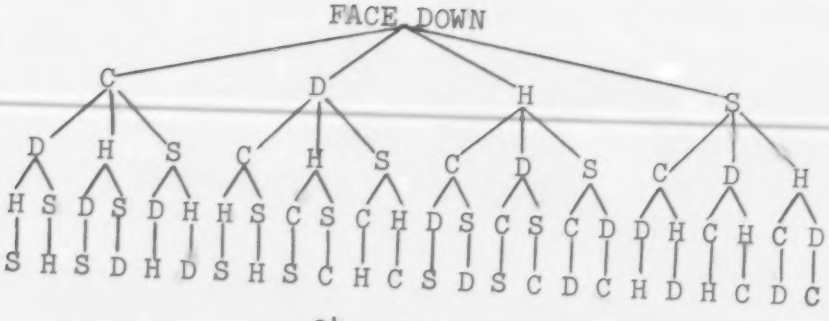
Q	D	N	P
1	1	1	2
1	0	2	2
0	4	0	2
0	3	2	2
0	2	4	2
0	1	6	2
0	0	8	2

LESSON 6B page 34

1st STOP	2nd STOP	3rd STOP
Chicago	Denver	San F.
Chicago	San F.	Denver
Denver	Chicago	San F.
Denver	San F.	Chicago
San F.	Chicago	Denver
San F.	Denver	Chicago



page 35



24 ways

page 36

Card 1	Card 2	Card 3	Card 4
C C C C C C	D D H H S S	H S D S D H	S H S D H D
D D D D D D	C C H H S S	H S C S C H	S H S C H C
H H H H H H	C C D D S S	D S C S C D	S D S C D C
S S S S S S	C C D D H H	D H C H C D	H D H C D C

-All-

page 37 1.

Group I	Group II	Group III	
1	1	7	
1	2	6 *	
1	3	5 *	
1	4	4	3 ways
2	2	5	
2	3	4 *	
3	3	3	

page 37 2. Various results will be obtained.

page 38

Pennies	Nickels	Dimes	Quarters
25	0	0	0
20	1	0	0
15	2	0	0
15	0	1	0
10	1	1	0
10	3	0	0
5	0	2	0
5	2	1	0
5	4	0	0
0	1	2	0
0	3	1	0
0	5	0	0
0	0	0	1

13 Grand children